Shrinking target sets for nonautonomous systems

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For an autonomous dynamical system $T\colon X \to X$ the shrinking target set $\mathscr{D}$ is the set of points in X whose orbits hit a sequence of balls $B(x_n, r_n)$, with $r_n \to 0$, infinitely many times. That is,

$$\mathscr{D} = \bigcup_{m=1}^{\infty} \bigcap_{n=m}^{\infty} T^{-n}\left(B\left(x_n, r_n\right)\right).$$

This type of sets are dynamical counterparts to sets of well-approximable numbers in Diophantine approximation.

A nonautonomous dynamical system consists of a sequence of maps $T_n\colon X \to X$ where iteration is defined as

$$T^n = T_n \circ T_{n-1} \circ \cdots \circ T_1.$$

In this talk we will consider two different kinds of problems related to shrinking targets: One regarding dimension and one regarding measure. For nonautonomous systems the story begins with work related to Cantor series expansions by Fishman, Mance, Simmons, and Urbański (regarding dimensions); and Sun and Cao (regarding measures). We will then talk about the more general setting of conformal (nonautonomous) iterated function systems in higher dimensions.