Universidade Federal de Alagoas – UFAL

Instituto de Matemática – IM

Plano de Atividades Acadêmicas 2020.2

Docente: José Anderson de Lima e Silva

SIAPE: 3140413

Atividades de Ensino:

Atividades de orientação, supervisão e outros em nível de Graduação e Pós-Graduação.

1. Atendimento extraclasse de turmas com mais de 40 alunos (2 turmas). **3 pontos**;

2. Monitoria (4 supervisões). **4 pontos**;

3. Iniciação científica (1 supervisão). **2 pontos**.

Atividades de Pesquisa:

Participação em projetos de pesquisa que tenham entre suas metas a divulgação dos resultados conforme Art. 10º, Incisos I, II, III. **20 pontos**.

1. Coordenador do Projeto de Pesquisa: Teoria Min-max aplicada as Superfícies Mínimas.

Escrita de artigos:

- The widths of the riemannian product of a unit circle with a unit two-sphere;
- Low min-max widths of the 3-dimensional real projective space.

Obs.: Continuação do projeto aprovado pelo PPGMAT. Durante este projeto já escrevemos dois *preprints* que estão submetidos: "A SHORT NOTE ABOUT 1-WIDTH OF LENS SPACES" e "THE FIRST AND SECOND WIDTH OF THE REAL PROJECTIVE SPACE".

Atividades Administrativas e de Representação:

1. Coordenação de monitoria. **6 pontos**;

2. Participação em reunião de colegiado ou CONSIM. **1 ponto**.

Pontuação total: 36 pontos.

Pontuação do PAA: 24 pontos.

A SHORT NOTE ABOUT 1-WIDTH OF LENS SPACES

MÁRCIO BATISTA* AND ANDERSON DE LIMA

Abstract. In this note we explore the nature of Lens spaces to study the first width of those spaces, more precisely we use the existence of a sharp sweepout associated to a Clifford torus to provide a simple and pretty application of the Willmore conjecture for the computation of the 1-width of Lens Spaces.

1. Introduction

In a remarkable work, [2], Almgren proved that:

The width of the round 3*-sphere is equal to* 4π *and the surface which reaches this number is an equatorial sphere.*

Later on, using, in a fundamental way, the symmetric structure of the round unit sphere in $\mathbb{R}^4$, Nurser motived by a question proposed by Marques & Neves, [14], computed the values of some widths of the round unit sphere, more precisely, he computed

$$\omega_1(S^3) = \omega_2(S^3) = \omega_3(S^3) = \omega_4(S^3) = 4\pi \text{ and } \omega_5(S^3) = 2\pi^2,$$

and he also gets some bounds to ω_9 and ω_{13}, see [16].

From the close relationship between $\mathbb{S}^n$ and $\mathbb{RP}^n$, do Carmo, Ros and Ritoré studied closed minimal hypersurfaces in $\mathbb{RP}^n$ and classified those hypersurfaces with index one, see [5]. More recently, Ramirez-Luna gets a similar result for hypersurfaces immersed in $\mathbb{CP}^n$ and using this classification and some tools from the min-max theory, she gets the value of the width of $\mathbb{RP}^n$ for n between 3 and 7, see [18]. Using another approach, the authors get the value of the first and second widths of $\mathbb{RP}^n$ for n in the same range as before, see [4].

Motivated by questions in the works of Guth [9], Gromov [8] and Marques & Neves [14], others works were done and the value of widths were computed, see for instance [1], [16], [3] and [6]. These works used strongly the symmetric structure of the target spaces.

2010 *Mathematics Subject Classification.* Primary: 53C42; Secondary: 49J35.
Key words and phrases. Minimal surfaces, p-width, Min-max theory.
*Corresponding author.

THE FIRST AND SECOND WIDTH OF THE REAL PROJECTIVE SPACE

MÁRCIO BATISTA* AND ANDERSON DE LIMA

Abstract. In this paper we deal with the first and second width of the real projective space $\mathbb{RP}^n$, for n in the range 4 to 7, in the setting of Almgren-Pitts min-max theory. In a recent paper, Ramirez-Luna, using a result due to do Carmo, Ritoré and Ros, computed the first width of the real projective spaces and, at the same time, we obtain optimal sweepouts which realize the first and second widths of those spaces.

1. Introduction

Liokumovich, Marques and Neves in [8] were interested in understand the volume spectrum of a given Riemannian manifold, $\{\omega_p(M)\}_p$, and they were able to prove a Weyl law for such non-linear spectral problem as conjectured by Gromov in [5] and so they obtained the more quantitative result after the works done by Gromov [5] and Guth [6]. Using also the volume spectrum, Marques and Neves provided a proof of one famous conjecture of Yau about minimal surfaces in the setting of Frankel property and in the same work, see [10, Section 9], they point out that would be interesting to compute the values of $\omega_p(M)$ in specific examples and verify whether such numbers are achieved by interesting minimal hypersurfaces. Therefore, in a recent work, see [2], the authors computed and provided some upper bounds for the widths on the real projective 3-space. More precisely, the authors proved:

For the p-widths of the 3-dimensional real projective space, $\mathbb{RP}^3$, we have

- $\omega_1(\mathbb{RP}^3) = \omega_2(\mathbb{RP}^3) = \omega_3(\mathbb{RP}^3) = \pi^2$;
- $\omega_9(\mathbb{RP}^3) \leq 4\pi$,

and the surface which realizes this number π^2 is the minimal Clifford torus $T^2/\{-x, x\}$ in $\mathbb{RP}^3$. Moreover, there exists a closed minimal surface Σ, with genus greater or equal to two, realizing the 4-width.

The computations of the widths in the above result relies on a number of crucial ingredients: the results of Almgren and Pitts [13], [16] and [7] about existence of closed embedded minimal hypersurfaces with index one and a construction of an optimal sweepouts. Moreover, the authors used an algebraic sweepout to provide an upper bound to the ninth width.

We note that the interest about the asymptotic behavior of the volume spectrum appears first in [5] and [6], and later on in [8]. In the first two works cited, the authors obtain an upper and lower bound of the p-width as a particular power of its parameter p. In [10], Marques and Neves developed the min-max theory and using the p-widths they proved a famous conjecture of Yau, as noted before.

We highlight that produce an optimal sweepout and so compute the width is a hard task and we have just a little works with this purpose. As example we cite one of the breakthroughs done by Marques and Neves, [9]. Roughly speaking, after them developed the necessary machinery, they introduced an optimal 5-sweepout associated to the minimal Clifford torus immersed in the

2010 *Mathematics Subject Classification.* Primary: 53C42; Secondary: 49J35.
Key words and phrases. Clifford hypersurfaces, Minimal hypersurfaces, Second width, Min-max theory.
*Corresponding author.