GENERALIZATION OF THE H_p-THEOREM IN A SPACE OF CONSTANT CURVATURE

Hilário Alencar **Antonio Gervasio Colares**

Let $x\colon M^n \to R^{n+1}$ be and isometric immersion of an oriented Riemannian manifold M^n with unit normal vector ν, mean curvature H and support function $p = -\langle x, \nu\rangle$. The H_p-Theorem says that if M^n is compact and

$$H_p = 1,$$

then $x(M^n)$ is a round sphere ([3]).

Here we announce two generalizations of the H_p-Theorem. The proofs will appear elsewhere.

Denote by Q_c^{n+1} an n-dimensional simply connected space of constant curvature c. If $p_0 \in Q_c^{n+1}$ we denote $r(\cdot) = d(\cdot, p_0)$ the distance function relative to p_0 and we write grad r for the gradient of r in Q_c^{n+1}. Let $x\colon M^n \to R^{n+1}$ be an isometric immersion of a Riemannian manifold M^n oriented by a unit vector ν. We call $X = S_c$ grad r the position vector of the immersion with respect to p_0, where $S_c(r) = r, \frac{\sin(r\sqrt{c})}{\sqrt{c}}$ or $\frac{\sinh(r\sqrt{-c})}{\sqrt{-c}}$, according $c = 0$, $c > 0$ or $c < 0$. The function $p = -\langle X, \nu\rangle$ will be called the support function of the immersion. We denote $\theta_c = \frac{d}{d_r}\, S_c(r)$.

Theorem 1. ([1]) *Let $x\colon M^n \to Q_c^{n+1}$ be an isometric immersion of a compact oriented Riemannian manifold M^n with mean curvature H and support function p. Then*

$$H_p - \theta_c$$

does not change sign if and only if $x(M^n)$ is a geodesic sphere.

A proof of this theorem is obtained from the following

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Lemma. *In the conditions of Theorem* 1*, if* Δ *is the Laplacian of* M^n*, then*

$$\frac{1}{2}\,\Delta\,\langle X,X\rangle = -c\,S_c^2\,|(\text{grad }r)^T|^2 - n\theta_c(H_p-\theta_c).$$

Theorem 2. *Let* $x\colon M^n \to S^{n+1}(c)$ *be an isometric immersion of a compact oriented Riemannian manifold* M^n *into the* $(n+1)$*-sphere of radius* $\frac{1}{\sqrt{c}}$*, with unit normal vector* ν*, mean curvature* $H>0$ *and support function* p*. If*

$$H = p,$$

then $x(M^n)$ *is a geodesic sphere.*

This theorem has been proved by G. Huisken ([2]) when the ambient space is the Euclidean space R^{n+1}.

References

[1] Alencar, H. and Colares, A. Gervasio, *A Characterization of Hyperspheres in Space Forms*, preprint.

[2] Huisken, G., *Asymptotic Behaviour for Singularities of the Mean Curvature Flow*, J. of Diff. Geom., 31, 285-299 (1990).

[3] Rotondaro, G., *On the* H_p*-Theorem for Hypersurfaces*, Comment. Math. Univ. Carolinae, 30 (2), 385-387 (1989).

Hilário Alencar
Departamento de Matemática
Univ. Federal de Alagoas
57000 Maceió, Alagoas

Antonio Gervasio Colares
Departamento de Matemática
Univ. Federal do Ceará
6000 Fortaleza, Ceará