GENERALIZATION OF THE $H_p$-THEOREM IN A SPACE OF CONSTANT CURVATURE

Hilário Alencar Antonio Gervasio Colares

Let $x: M^n \to R^{n+1}$ be an isometric immersion of an oriented Riemannian manifold $M^n$ with unit normal vector $\nu$, mean curvature $H$ and support function $p = -\langle x, \nu \rangle$. The $H_p$-Theorem says that if $M^n$ is compact and

$$H_p = 1,$$

then $x(M^n)$ is a round sphere ([3]).

Here we announce two generalizations of the $H_p$-Theorem. The proofs will appear elsewhere.

Denote by $Q^{n+1}_c$ an $n$-dimensional simply connected space of constant curvature $c$. If $p_0 \in Q^{n+1}_c$ we denote $r(\cdot) = d(\cdot, p_0)$ the distance function relative to $p_0$ and we write $\text{grad} r$ for the gradient of $r$ in $Q^{n+1}_c$. Let $x: M^n \to R^{n+1}$ be an isometric immersion of a Riemannian manifold $M^n$ oriented by a unit vector $\nu$. We call $X = S_c \text{grad} r$ the position vector of the immersion with respect to $p_0$, where $S_c(r) = r, \frac{\sin(r\sqrt{c})}{\sqrt{c}}$ or $\frac{\sinh(r\sqrt{-c})}{\sqrt{-c}}$, according $c = 0, c > 0$ or $c < 0$. The function $p = -\langle X, \nu \rangle$ will be called the support function of the immersion. We denote $\theta_c = \frac{d}{dr} S_c(r)$.

Theorem 1. ([1]) Let $x: M^n \to Q^{n+1}_c$ be an isometric immersion of a compact oriented Riemannian manifold $M^n$ with mean curvature $H$ and support function $p$. Then

$$H_p - \theta_c$$

does not change sign if and only if $x(M^n)$ is a geodesic sphere.

A proof of this theorem is obtained from the following

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Lemma. In the conditions of Theorem 1, if $\Delta$ is the Laplacian of $M^n$, then
\[
\frac{1}{2} \Delta \langle X, X \rangle = -c R^2 - n\theta_c (H_p - \theta_c).
\]

Theorem 2. Let $x: M^n \to S^{n+1}(c)$ be an isometric immersion of a compact oriented Riemannian manifold $M^n$ into the $(n+1)$-sphere of radius $\frac{1}{\sqrt{c}}$, with unit normal vector $\nu$, mean curvature $H > 0$ and support function $p$. If
\[
H = p,
\]
then $x(M^n)$ is a geodesic sphere.

This theorem has been proved by G. Huisken ([2]) when the ambient space is the Euclidean space $R^{n+1}$.

References


Hilário Alencar
Departamento de Matemática
Univ. Federal de Alagoas
57000 Maceió, Alagoas

Antonio Gervasio Colares
Departamento de Matemática
Univ. Federal do Ceará
6000 Fortaleza, Ceará